

The Imposition of Empirical Acceptability Conditions on Sequences of Single Measurements

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Here, some difficulties resulting from the application of any empirical acceptability conditions on sequences of single measurements are investigated. In particular, the often used acceptability requirement that each single measurement be made under the “same conditions” is discussed. In quantum mechanics, this means that each single measurement is made of the same physical quantity on a system in an ensemble of identically prepared systems.

One of the resultant difficulties is that such an application leads to an infinite regression of sequences of single measurements. That is, it does not account for the fact that an observer must start the process of measurement or knowledge acquisition. Furthermore, it is seen that there are some basic sequences of single measurements for which an observer can not possibly know at the outset that the “same condition” requirements are satisfied. These include those measurements by which the homogeneity of space-time is tested.

The possible relevance of these difficulties to physics is shown by first considering two possibilities of avoiding these difficulties. One is that the “same condition” requirements can be given the weaker interpretation that there be no physical principle forbidding an observer from knowing in terms of limit empirical means, that they are satisfied at the outset of any sequence. This gets rid of the infinite regression problem as it does not mean that an observer must know in fact that these requirements are satisfied.

The other possibility is that if physics does not forbid one in principle from measuring an expectation value in an arbitrarily small time interval then both the basic sequence as well as those by which one knows the “same” requirements are satisfied can be relegated to arbitrarily small time intervals. As far as physics is concerned, then the epistemological difficulties while existing in these small intervals, do not exist for other times, or almost all time.

It is then shown that quantum mechanics, as distinct from classical mechanics, and the special relativity require that an infinite time interval is necessary to measure, as a limit mean, any expectation value. Thus physics denies both the above possibilities as it forbids an observer from knowing even in principle, by any finite time that the “same” requirements are satisfied. Also, physics forbids the relegation of the epistemological problems to arbitrarily small time intervals.

I. Introduction

In science in general and particularly in quantum mechanics, contact between theory and experiment is made by means of limit empirical means obtained from infinite sequences of single measurements. These limit means which are to be compared with an expectation value are obtained as the limiting value as $N \rightarrow \infty$ of the sequence of empirical means, M_N , associated with a sequence of single measurements. M_N is the average of the results of the first N single measurements.

It is also quite clear that any arbitrary sequence of single measurements is not an acceptable measurement of an expectation value. There are some requirements which a sequence must satisfy in order to be acceptable. Now the requirements which are usually applied and would seem to be quite natural involve the concepts of “same” and “independence”. That is, any acceptable sequence is a sequence of “independent” single measurement made under the

“same conditions”. By “independence” it is meant that the single measurements in a sequence must be statistically independent of one another. The outcome of any single measurement must neither affect nor be affected by the outcome of any other single measurement.

By “same conditions” is meant an objective concept rather than a subjective concept in the following sense. Any sequence of single measurements made with a piece of equipment which malfunctioned intermittently would be discarded. For even though the piece of equipment might “look the same” to an observer for each single measurement in that he did not notice the intermittent fluctuation of a meter needle on the equipment, the meter reading may be quite relevant. If the observer later discovered that the meter reading had been fluctuating during the sequence, he would discard the sequence on the grounds that each single measurement was not made under the “same” conditions.



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These requirements of “same” and “independence” appear to play a rather basic role in science. For example, in quantum mechanics, any sequence of single measurements from which an expectation value, $\text{Tr} \rho 0$, is obtained, is usually required to satisfy the following conditions: 1) The ensemble of systems, described by ρ , on which the measurement sequence is made, must be identically prepared or prepared under the “same relevant¹ conditions”; 2) each single measurement must be made of the “same observable”; and 3) each single measurement must be “independent” of the others.

As another example, consider the basic fact that one always performs at most a finite sequence of single measurements and must relate the obtained empirical mean M_N to a limit expectation value, μ . The statistical methods by which one says that M_N is “close to” or “a good approximation for” μ also usually involve the concepts of “same” and “independence”. Consider for example the probabilistic expression

$$P\left(M_N - \frac{q\sigma_N}{\sqrt{N}} < \mu < M_N + \frac{q\sigma_N}{\sqrt{N}}\right) = P(q) \quad (1)$$

where M_N and σ_N are the sample mean and standard deviation and $P(\)$ denotes a probability².

Now the usually accepted meaning of this and similar probability theoretic expressions is that if the finite sequence of N single measurements, considered as a unit or single measurement, is performed an infinite number of times under the “same conditions” such that each unit measurement is “independent” of the others, then the fraction of intervals $[M_N - q\sigma_N/\sqrt{N}, M_N + q\sigma_N/\sqrt{N}]$ which contain μ is given by $P(q)$ ^{2,3,4}.

The purpose of this paper is to examine some limitations and consequences of imposing these “same and independence” requirements on arbitrary sequences of single measurements. In particular, this work will mainly be concerned with the “same” requirements as the conclusions reached for these requirements apply also to the “independence” requirements. By the abbreviation “same” requirements’ will be meant the requirements that the

single measurements in a sequence all are made under the same conditions. In quantum mechanics, this means that each single measurement of a sequence must be made of the same observable on a system in an ensemble of identically prepared systems.

In Section II, some limitations of the “same” requirements are considered. It is shown that if these requirements mean that an observer must in fact know that they are satisfied before beginning a contemplated sequence of single measurement, then imposition of these requirements leads to an infinite regression. For in this case, any sequence of single measurements must always be described in terms of earlier completed sequences by which an observer knows that these “same” requirements are satisfied. Thus imposition of these requirements in the sense noted above has the consequence that it does not allow an observer to start the process of measurement or knowledge acquisition.

Furthermore, in the same section it is seen that an observer *must* consider sequences of basic single measurements for which he can not know that these requirements are satisfied. These measurements are essentially those by which one determines whether or not space-time is homogeneous or whether or not an invariance principle holds for space-time displacements⁵.

The arguments given so far lose their force for physics if either of two possibilities are allowed. One possibility is that the meaning of the “same” requirements can be relaxed to be that an observer need not *in fact* know at the outset that these requirements are satisfied but only that they must be *in principle* knowable in terms of limit empirical means. That is, there must be no physical principle which forbids one from knowing in terms of limit empirical means or expectation values, that these requirements are satisfied at the beginning of any sequence. The other possibility is that if one can in principle carry out all the basic measurements including those by which one knows that the requirements are satisfied for any given sequence in

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¹ J. M. JAUCH, *Helv. Phys. Acta* **37**, 293 [1964].

² S. S. WILKES, *Mathematical Statistics*, John Wiley and Sons, Inc. New York 1962, Chaps. 1, 8, 10.

³ W. FELLER, *An Introduction to Probability Theory and its Applications*, Vol. I, 2nd Edition, John Wiley and Sons, Inc. New York 1965, Introduction and Chap. I.

⁴ R. VON MISES, *Mathematical Theory of Probability and Statistics*, Edited by H. GEIRINGER, Academic Press, New York 1964, Chap. I.

⁵ E. WIGNER, *Nuovo Cim.* **3**, 517 [1956]; R. F. HOUTAPPEL, H. VAN DAM, and E. WIGNER, *Rev. Mod. Phys.* **37**, 595 (1965).

an arbitrarily small time interval, or in a countable set of such intervals, then the difficulties discussed above, are restricted to these intervals only. For the remaining times one already knows how, in principle at least, to construct sequences which satisfy the "same" requirements and the problems discussed do not arise.

It is clear that these possibilities are equivalent to the requirement that there be no physical principle which forbids one from measuring a limit empirical mean or expectation value in an arbitrarily small time interval. This problem is examined in Section III where it is seen that quantum mechanics (in contrast to classical mechanics) and the finite signal velocity of relativity require that an infinite time interval is necessary to measure even one expectation value.

Thus even the weaker meaning attributed to the "same" requirements does not remove the limitations already discussed. Furthermore, the above argument shows one can not remove the basic measurement sequences for which one can not know that these requirements are satisfied to arbitrarily small time intervals but that, like all other infinite sequences, they must in principle occupy a infinite time interval. Thus the problems of how one describes these measurement sequences as well as how starts the measurement process may be relevant to physics in that quantum mechanics with the finite signal velocity of relativity require that these problems be solved.

Section IV is a discussion of several aspects of these results. Among other things it is noted that the conclusions reached here can be extended to include any empirically decidable property which is made a condition of acceptability for sequences of single measurements. Also this work stresses the importance of the consideration of the beginning of the process of measurement or knowledge acquisition. For instance, it is of importance to find minimum conditions of acceptability for any sequence of single measurements. The minimum ergodic conditions of probability theory⁶ are briefly discussed.

Before beginning the discussion of Section II, it is necessary to stress several important points. First of all, the arguments given in this paper are not limited to the "same" and "independence" requirements. They apply to any requirements or con-

ditions placed on a sequence which imply prior empirical knowledge about each single measurement of the sequence. Thus they would apply for any probability theoretic properties imposed as requirements on a sequence of single measurements. A closely related point is that if an observer can always know how to construct sequences which satisfy the "same" requirement, then he can always construct sequences of single measurements with arbitrary properties by selecting single measurements out of different sequences in some order. For example, one can construct a sequence of single measurements of the spin projection P_{up} in quantum mechanics with an arbitrary probability distribution by using two different preparation apparatuses associated with states ρ_1 and ρ_2 , in conjunction with a Stern Gerlach apparatus. Then if $\text{Tr} \rho_1 P_{\text{up}} \neq \text{Tr} \rho_2 P_{\text{up}}$ one can construct a sequence of single measurements which does not satisfy the "same" requirements (preparation of the ensemble under the same relevant conditions) merely by using the two preparation apparatuses in some order as preparation apparatuses for the sequence of single measurements.

Finally, the reason that the "same" requirements play such a basic role in this discussion is that only from sequences which satisfy the "same" requirements can one know empirically the properties of a single measurement. This was discussed in other work⁶ where it was shown that for general ergodic sequences of single measurements, the empirical properties obtained apply only to the limit ensemble of single measurements and not to each individual measurement. That is, only the limit ensemble probability measure was empirically measurable and it implied nothing about the measure associated with each single measurement. Only if the "same" requirement was satisfied for a sequence did the properties of the limit ensemble apply to the single measurements in that the single measurement probability measure coincided with the limit ensemble measure.

II. The "Same" Requirements

A) Infinite Regression

Let us first consider what happens if the "same" requirements are interpreted to mean that, before beginning any sequence of single measurements in quantum mechanics, an observer must in fact know

⁶ P. BENIOFF, J. math. Phys. 9, 514 [1968].

that these requirements are satisfied by the empirical procedure to be used. Let x_j with $j = 1, 2, \dots$ label a set of arbitrarily large disjoint space-time regions within each of which a single measurement is to be made. Then the above requirements applied to this set or sequence of single measurements mean that the systems prepared by some procedure, one system within each region x_j , must in fact be known to be prepared under the same conditions. Similarly, the measurement procedure used must be known to measure the same observable within each region x_j . This immediately brings up the question of how an observer is to know that the preparation and measurement procedures to be used for a contemplated sequence will indeed satisfy the "same" requirements for each x_j . One possible answer is provided by the basic axioms of quantum mechanics⁷. The relevant axioms state that two states ϱ_1 and ϱ_2 are the same if the expectation values of ϱ_1 and ϱ_2 are the same for all projection operators i.e., $\text{Tr } \varrho_1 P = \text{Tr } \varrho_2 P$ for all P . Similarly, two projection operators, P_1 and P_2 , are the same if the expectation values taken with respect to all states are the same, i.e., if $\text{Tr } \varrho P_1 = \text{Tr } \varrho P_2$ for all state ϱ .

This immediately suggests the following method to tell if the preparation apparatus is satisfying the "same preparation conditions" requirement: Prepare, using the apparatus, large ensembles of systems *within* region x_1 and using many different measurement apparatuses representing many observables, measure the expectation values with respect to all the observables within x_1 . Then translate the preparation apparatus and the many measurement apparatuses to region x_2 and repeat the procedure of preparing a large ensemble and measuring many different expectation values. If the appropriate expectation values compared between the regions x_1 and x_2 were pairwise equal, then the observer would know that the first system used in the first single measurement in x_1 for the original sequence was prepared under the same conditions as that used in the second single measurement in x_2 .

This procedure can then be extended to all the other regions x_3, x_4, \dots and exactly the same procedure will tell whether systems are being

prepared under the same conditions in x_3, x_4, \dots . An entirely similar procedure can be used to tell if the measurement apparatus used, measures the same projection operator in each of the regions x_j . In this case, large ensembles are prepared within each region x_j by many state producing apparatuses and, using the given measurement apparatus, expectation values for each preparation apparatus are determined. Again, if the appropriate expectation values are pairwise equal for any regions x_j, x_k then the observer knows that the measurement apparatus measures the same projection operator in each of the single measurements of the original sequence, in each region x_j . By well known methods, this then extends to all observables⁷.

It is worthwhile to note at this point, several aspects of the discussion presented so far. First of all, the procedure discussed does not imply that the ensembles produced in each region x_j by the contemplated preparation apparatus consist of the same type of systems. For example, the ensemble prepared in x_1 can consist of protons, neutrons, stones, etc. produced in some given proportion. However, whatever systems are produced in x_1 , must be produced in x_2, x_3, \dots and in the same proportions. In fact, this is just what is obtained following the procedure just described.

Another point is that for mixed states, such as this ensemble of protons, etc., one must be sure that the systems prepared or selected for the original sequence of single measurements must be prepared under the same lack of bias as was used for the single measurements within each region x_j . That is, if protons, neutrons, etc. were prepared in each of the regions x_j , then the observer must ensure that the single systems selected for each single measurement in x_1, x_2, \dots are also sometimes protons, sometimes neutrons. Equivalently, the method of selection must be random or independent at the type of system prepared.

Finally, it has been noted¹ that for the measurement of the expectation value of any observable O one need not require that each system be prepared under the same conditions but only that each system be prepared under the same relevant (with respect to O) conditions. This weaker requirement would greatly reduce the number of expectation values which must be measured within each region, x_j . However, use of this weaker requirement does not change any of the conclusions of this work and

⁷ G. W. MACKEY, *Mathematical Foundations of Quantum Mechanics*, W. A. BENJAMIN, Inc. New York 1963 Chap. II; J. VON NEUMANN, *Mathematical Foundations of Quantum Mechanics*, Translated by R. T. BEYER, Princeton University Press, New Jersey, 1955, Chap. IV; H. EKSTEIN, *Ergeb. Exakt. Naturwiss.* **37**, 150 [1965].

so will be neglected. Similarly problems associated with the fact that the procedure suggested by quantum mechanics requires measurement of the expectation values of an infinite number of projection operators and states within each region x_j will be neglected here.

The main difficulty arises when account is taken of the fact that the "same" requirements are to apply to *all* sequences of single measurements. This means that they also apply to each of the many sequences occurring within each region x_j by which an observer knows that the "same" requirements are satisfied for the original sequence. In order to know, for any one of these sequences, that the "same" requirements are satisfied, the same procedure requires that an observer must consider each much smaller region $x_{\alpha j}$ ($\alpha = 1, 2, \dots$) which contains a single measurement and repeat the whole process by performing many sequences within each smaller region. Then, from the results of all these sequences, he would know that each single measurement made in each $x_{\alpha j}$ for the appropriate sequence was made under the same conditions. But then in order to know that the same requirements are satisfied for these sequences, an observer must make still more sequences of single measurements within still smaller intervals, etc.

Thus aside from the difficulty of having to make sequences of single measurements in smaller and smaller space time regions, the above argument leads to an infinite regression of sequences of single measurements. For if one must in fact know that the "same" requirements are satisfied for every sequence of single measurements, then every sequence of single measurements is always described in terms of other sequences of single measurements. As a result, an observer can not start the process of measurement or knowledge acquisition. The reason is that at the beginning of the process, he has not made any sequences of single measurements and consequently can not know that the "same" requirements are satisfied. For this reason, these "same" requirements, at least, if they are interpreted as requiring in fact prior knowledge of their satisfaction, can not be part of the description of the measurement of any expectation values.

So far the discussion has been based on the procedure suggested by the axioms of quantum mechanics by which an observer can know that the "same" requirements are satisfied. However, it is

worth-while to discuss an alternate procedure which, although related to the previous one has the advantage that it is closer to what is actually done in the laboratory. Furthermore, besides resulting in the same infinite regression problem, it also shows clearly the existence of basic sequences of measurements in physics for which one can not know that the "same" requirements are satisfied.

Consider again the problem of how one knows that for any two single measurements, the preparation apparatus prepares two systems under the same conditions and the measurement apparatus measures the same physical quantity each time. It is clear that such knowledge involves knowing which quantities are relevant and which are irrelevant to the apparatuses involved. For example, if the preparation apparatus is temperature or light sensitive, then preparation under the same conditions requires that for each single measurement the temperature and light intensity be the same. For example, this can actually be ensured by attaching a thermometer and a photocell to the preparation apparatus and requiring that the readings be the same for any pair of single measurements. Furthermore, by attaching more meters to the preparation apparatus one can ensure that many other relevant conditions are the same for both preparations.

Now suppose that as before, the "same" requirements are interpreted to mean that at the outset of any given sequence of single measurements, one must know what conditions e.g. temperature, etc. must be kept constant for both the preparation and measurement apparatuses. Then it appears that one ends up in the same infinite regression difficulty as before. The reason is that in order to know which conditions are relevant and which are not, one must have already performed many sequences of single measurements in which for each sequence some external condition was kept constant but the condition was different for different sequences. By comparing the expectation values obtained from the different sequences for different values of the same condition (e.g., different temperatures or different point colors on the apparatuses) one can map out which conditions are relevant and which are irrelevant to knowing whether the "same" requirements are satisfied or not for the given original sequence. But again, if the "same preparation condition" and "same physical quantity"

requirements apply also to each of these sequences then still earlier groups of sequences must be carried out, etc. Thus with this alternate procedure, one still ends up in the same infinite regression difficulty. Also, as before, it does not allow one to start the process of measurement or knowledge acquisition.

B) Homogeneity of Space-Time

The discussion given so far suggests a variant of the above method which, at first, appears to get rid of this difficulty, but in fact shows that there are fundamental measurements for which one can not know that these "same" requirements are valid. Suppose an observer at the outset places many meters, each one sensitive to a different physical condition or aspect on both the measurement and preparation apparatuses and then requires that they all give the same readings for each single measurement. Furthermore, he does this without any prior knowledge about which meter readings are relevant and which are irrelevant. Thus it would appear that by regulating as many conditions as possible, irrespective of whether they are relevant or not, the observer would automatically ensure that the "same preparation condition" and "same physical quantity" requirements were satisfied.

However, this is not possible for the basic reason that there is at least one meter which *must* give different readings for each single measurement of any sequence. This is the meter which labels the space-time region within which each single measurement occurs. This is a consequence of the fundamental fact that any two distinct events must occupy separate space-time regions. Thus we see that before we can know that "the same preparation condition" and "same physical quantity" requirements are satisfied, we *must* already know that the labels of the space-time regions associated with each single measurement are irrelevant variables.

It might be immediately objected that because of the homogeneity of space-time or the validity of an invariance principle for space-time translations⁵ one knows that these labels are irrelevant variables. However, this implies prior knowledge of the homogeneity of space-time. Since this empirically decidable⁵ this objection is met by considering those sequences of single measurements by which the homogeneity of space-time is decided.

It is clear that for such basic sequences of single measurements, one can not possibly know that the "same preparation condition" and the "same physical quantity" requirements are satisfied. For the above argument shows that in order to know this, one must already know that the space-time labels of each single measurement are irrelevant variables. But this is something which is to be empirically decided by these basic measurement sequences and one can not possibly know the results of these sequences before they are carried out.

One method which avoids this problem is to just arbitrarily assume that space-time labels of each single measurement are irrelevant. For example, the systems of an ensemble prepared by some given procedure will be arbitrarily defined to be prepared under the same conditions. Then one could hope that as more and more measurements are made, he would find out whether the assumption is empirically consistent with other data.

Aside from operational problems as to how this procedure is to be defined⁸, the main difficulty with such an arbitrary definition is that one must be quite careful about what properties are attached to the measurement process. For how the process is described, especially for such basic measurements as those discussed, may have nontrivial consequences for the knowledge acquisition process and possibly even for physics.

One way to see this is to note that how sequences of single measurements are described affects the basic meaning or implications of all contacts between theory and experiment. For one thing, the basic meaning of such contacts or of a comparison between theory and experiment is stronger for sequences which satisfy the "same" requirements than for those which do not⁶. The reason is that for sequences which satisfy the "same" requirements, the limit empirical means obtained apply both to the whole ensemble of single measurements as well as to each single measurement. However, for sequences which do not satisfy these "same" requirements the limit empirical means apply to the whole ensemble only and not to the single measurements⁶. This means that, for sequences which satisfy the "same" requirements, whatever one learns from a comparison between theory and experiment applies to *both* the ensemble of single

⁸ H. EKSTEIN, Phys. Rev. **153**, 1397 [1967].

measurements as well as to each single measurement. But for sequences which are not known to satisfy the "same" requirements, whatever one learns from such a comparison applies to the whole ensemble only. The comparison gives no information, theoretical or experimental about the single measurements.

If these aspects are coupled with the complex epistemological relationships between various physical measurements, it is seen that the arbitrary assumption that the "same" requirements are satisfied could well affect the physical consequences attributed to the results of measurements. For example, the strong feedback property of the knowledge acquisition process coupled with the aspects discussed might lead to the situation in which the arbitrary assumption might be supported by the empirical data. On the other hand, if the assumption was not made, then exactly the same empirical data, which are now weaker in their implication or meaning, might say nothing about whether the procedure used satisfies the "same" requirements and might even yield some new physics.

III. Possible Relevance for Physics

A) The Relation Between the Epistemological Problems and Physics

So far in the discussion, the problems such as how the process of measurement or knowledge acquisition is started and how one is to describe the basic sequence of single measurements by which the homogeneity of space-time is tested, have been discussed as mainly epistemological problems. Essentially, nothing was said about whether these problems have any relevance to physics. In this section, we would like to suggest that these problems may be relevant to physics in that physics does have something to say about them.

To see this, it should be recalled that the argument that one could not start the process of acquiring knowledge if the "same" requirements were applied to sequences of single measurements depended on the premise that these requirements meant that an observer must in fact know how to construct sequences of single measurements made under the same conditions. This argument loses its force, however, if one gives a different meaning to these requirements. This is, that, rather than an observer knowing in

fact that they are satisfied at the outset, they can be interpreted to mean only that an observer must be able in principle to know they are satisfied. That is, physics must not forbid an observer to know, in terms of limit empirical means, by any finite time, that each single measurement in a sequence is made under the same conditions. With this interpretation, then, as far as physics is concerned, an observer can start the measurement process without knowing any limit empirical means. In this case, the argument about the resultant infinite regression and how to start the measurement process, while still of relevance for epistemology, is not relevant for physics.

A similar situation exists regarding the problems of how to describe or construct basic sequences of single measurements by which one tests an invariance principle for space-time displacements. To see this, consider what happens if physics allows the determination of a limit empirical mean (i. e. the performance of an infinite sequence of single measurements) within an arbitrarily small time interval. In this case, these and any other basic sequences of single measurements can, as far as physics is concerned, all occur in one or even in many arbitrarily small time intervals scattered over the time axis. For all other times, which can consist of most of the time, an observer then can know the results of these basic measurements (in terms of limit empirical mean). In particular, he can already know that space-time is homogeneous and he can know how to construct sequences of single measurements which satisfy the "same preparation condition" and "same physical quantity" requirements. In this case, while the epistemological problems still exist, they are important only in these arbitrarily small time intervals and are thus irrelevant to physics.

Now the point we want to make is that the epistemological problems discussed above may be relevant to physics because quantum mechanics, as distinct from classical mechanics, with the finite signal velocity of relativity requires that an infinite time interval is necessary to measure even one limit empirical mean, or expectation value. Thus the basic sequences of single measurements and the associated problems can not be relegated to arbitrarily small time intervals.

B) Classical Mechanics

Let us first discuss classical mechanics with respect to the time interval necessary to obtain any

limit empirical mean. As is well known, in classical mechanics, it is not even necessary to discuss sequences of single measurements because one single position and one single momentum measurement is, in principle, sufficient to determine completely the state or trajectory of the system. However, limit means and sequences of single measurements often enter because an observer in fact does not, or in practice can not make position and momentum measurements on each system. Also he may not have in fact made sufficiently exact position and momentum measurements to obtain an exact trajectory of a system. In this case, these measurements must be repeated over and over again either on "identically prepared" systems or on the "same" system in order to obtain both a probability distribution of trajectories as well as the expectation value of the position and momentum of a single system.

The main point of interest here is that in classical mechanics both the preparation of a single system, the correction for any possible influence a prior measurement may have had, and the measurement of any physical quantity can, in principle, be done within an infinitesimal space-time volume. This means that an infinite sequence of single measurements necessary for the determination of an expectation value can in principle be carried out in a finite and arbitrarily small space-time volume. In particular, the time interval in which an expectation value is measured, can according to classical mechanics be arbitrarily short. In this case, the problems discussed about how one starts the knowledge acquisition process and how one describes basic sequences of single measurements, although interesting epistemologically, are irrelevant to Physics.

C) Quantum Mechanics

The situation is quite different if quantum mechanics is the basic objective physical theory. In this case, quantum mechanical states, ϱ , replace classical trajectories as the basic description of physical systems. Furthermore, expectation values play a fundamental role as they are the only means by which states and observables are operationally

defined in the theory as well as the only points of contact between theory and experiment.

As is well known, an expectation value is operationally defined as the limit empirical mean obtained from an infinite sequence of single measurements. Here, in distinction to classical mechanics, the preparation and interaction of a single system with a measurement apparatus, i.e., a single measurement, requires a finite spacetime volume. Since the state description applicable to each system is a wave packet or a mixture of wave packets, we see that the space-time volume required in the preparation must be at least as large as that occupied by most of the packet. If this is not true then the preparation apparatus will chop off appreciable parts of the packet and thus prepare a different state. Or to put this another way, the preparation of each system of an ensemble described by ϱ must occupy a space-time volume at least as large as that occupied by ϱ . Otherwise the preparation apparatus will chop off appreciable parts of ϱ and produce another state ϱ' .

Now if one requires that there be essentially no interference between the single measurements then each single measurement must occupy a separate space-time volume. For if the space-time volumes of the single measurements were to overlap then the states of each system in the ensemble would overlap with one another and cause interference. If this requirement is then coupled with the fact that an infinite number of single measurements is required to determine an expectation value, then it is seen that quantum mechanics requires an infinite space-time volume in order to measure an expectation value.

Another way this can be seen is by consideration of the measurement apparatus. As is well known, the measurement apparatus must be of finite size⁹⁻¹¹. For if it were allowed to become arbitrarily small, it would, in the limit of infinite small size, fail as a measurement apparatus. For one thing, the detection efficiency for particles becomes very small especially if the size of the apparatus is much smaller than the wavelength of the system with which it interacts. These detection problems become

⁹ E. WIGNER, Z. Physik **131**, 101 [1952]; H. ARAKI and M. YANASE, Phys. Rev. **120**, 622 [1960].

¹⁰ M. M. YANASE, Amer. J. Phys. **32**, 208 [1964].

¹¹ G. LUDWIG, Solved and Unsolved Problems of the Measurement Process in Quantum Mechanics, Translated by E. Wilip from Werner Heisenberg and Contemporary Physics, Editor F. BOPP, F. Vieweg and Sohn, Braunschweig 1961, pp. 150-181.

very acute for systems of very low energy¹² and very long wavelength^{5,13}. Now, if the measurement apparatus is required to be macroscopic, then it occupies a finite space region. Since relativity requires that any signal take a finite time to cross a finite space region, we see that in order for the relevant parts of the measurement apparatus to act coherently and signify that an interaction has occurred, a finite time interval must elapse. Thus it is seen that whether one considers the preparation apparatus or the measurement apparatus, quantum mechanics requires that a single measurement must occupy a finite space-time volume. As a result, an infinite sequence of single measurements, which is necessary to measure an expectation value, must then occupy an infinite space-time volume.

From the arguments given so far, one only need require that any one dimension of the space-time volume be infinite in order to measure an expectation value. For instance, if the sequence of single measurements is purely time distributed, it will occupy a finite space region but take an infinite amount of time to carry out. If the infinite sequence is purely space distributed (many copies of the preparation and measurement apparatus distributed in space) then it will require an infinite amount of space but require a finite amount of time to carry out.

However, we maintain that it *always* requires in principle an infinite time interval to make an infinite sequence of single measurements irrespective of their space distribution. For if the infinite sequence covers an infinite space volume then relativity requires that the signal carrying the results of the arbitrarily distant single measurements take an arbitrarily long time to reach the observer. As a result there is no possible way an observer can perform an infinite sequence of single measurements in a finite time.

Because this point violates the "primitive causality" postulate¹⁴, it is necessary to examine it further¹⁵. There are several possible ways an

observer can perform an infinite space distributed sequence of single measurements. They can be arranged so that with respect to the local time of each single measurement, they all occur at the same time and, in this case, the signals giving the results of each measurement will be arriving throughout an infinite time interval. On the other hand, they can be arranged so that the more distant single measurements occur earlier with respect to their local time. Then all the signals from the infinite set of measurements arrive in a finite time interval. Leaving aside the problem of how an observer is to extract an infinite amount of information from a signal in a finite time, one sees that in this case the sequence of single measurements then extends back into the infinite past.

However, just as fundamental as a discussion of these aspects is the fact that just to make arrangements for an infinite set of measurements covering an infinite space region requires in principle an infinite amount of time. For such arrangements which for the above discussion, require a synchronization of the clocks attached to each single measurement, also require signals to be sent between the various single measurements and the observer and thus require an infinite time interval. Thus one sees that when the finite signal velocity is taken into account, an observer will *always* require an infinite time interval to perform an infinite sequence of measurements irrespective of whether or not they are spread over an infinite space volume¹⁶.

One consequence of this is that for any sequence of measurements started in the finite past one can not, by any finite time t , no matter how far into the future it is, obtain an expectation value. Only for those measurement sequences begun in the infinite past, or equivalently, without a beginning¹⁷ can one in principle measure an expectation value by any time t . Now quantum mechanics makes no statement about the possibility or impossibility of sequences of single measurements begun in the infinite past. However, this possibility will be

¹² L. BRILLOUIN, *Science and Information Theory*, 2nd Edition, Academic Press, Inc. New York 1962, Chap. 14.

¹³ H. BORCHERS, R. HAAG, and B. SCHROER, *Nuovo Cim.* **29**, 148 [1963].

¹⁴ R. HAAG and B. SCHROER, *J. Math. Phys.* **3**, 248 [1962]. According to this reference this postulate says in effect that one can determine the state of a system by expectation value measurements spread out arbitrarily in space but occupying an arbitrarily small time interval.

¹⁵ W. C. DAVIDON and H. EKSTEIN, *J. Math. Phys.* **5**, 1588 [1964].

¹⁶ In classical mechanics, this relativity requirement of a finite signal velocity is irrelevant because one can, in principle make an infinite number of single measurements in a finite spacetime volume.

¹⁷ B. RUSSEL, *Our Knowledge of the External World*, Allen and Unwin Ltd., London 1922, Lecture VI.

ignored here if for no other reason than the fact that all scientific measurements were begun in the finite past. That is, the process of knowledge acquisition has a beginning in the finite past. Thus the above discussion has shown that quantum mechanics and relativity require that an infinite time interval is necessary to measure an expectation value. Coupled with the exclusion of the infinite past as a starting point this means that an observer can not, by any finite time, no matter how far into the future, have measured even one expectation value as a limit empirical mean.

The consequences of these results for the problems discussed earlier can be easily seen. For one thing, even the weaker implication of the "same" requirements, that an observer need not in know fact that they are satisfied at the outset of a sequence but only that physics must not forbid this knowledge (in terms of limit empirical means) does not hold. For quantum mechanics and relativity forbid one from measuring even one limit empirical mean or expectation value by any finite time. Thus an observer can not even in principle know, in terms of limit means, at the outset of any given sequence, whether the single measurements are made under the same conditions or not. So it appears that the infinite regression problem mentioned earlier and the problems of how one starts the process of acquiring knowledge, may be relevant to physics. The reason is that quantum mechanics and relativity in contrast to classical mechanics, deny even the weaker escape from these problems.

Similarly, it appears that the problem of how to describe those fundamental sequences of single measurements by which the homogeneity of space-time is tested, may be relevant to physics. For quantum mechanics and relativity forbid the relegation of these sequences and the problems discussed to arbitrarily small-time intervals. In fact, they require more in that such sequences, in common with any other sequences begun at a finite time and yielding an expectation value, can not be completed by any finite time. Thus, the problems of how such sequences are described can not be ignored for most times, as is the case for classical mechanics, but exists for all finite times. Thus it appears that quantum mechanics and relativity require that this problem be considered.

IV. Discussion

There are some additional aspects of the discussion given so far that are worth noting. First of all, as is discussed in detail elsewhere¹⁸, the conclusions of this paper regarding the infinite regression or impossibility of starting the process of knowledge acquisition can be extended to any empirically decidable property. For any such property, the prior knowledge of which is made a condition or requirement of acceptability for a sequence of single measurements, is validated (or refuted) by other sequences of single measurements. Since any such property is also a condition of acceptance of these other sequence, knowledge that the property exists implies still other sequences, etc. Thus the same infinite regression occurs as well as the fact that making any such property part of the basic conditions of acceptability for a sequence does not allow the knowledge acquisition process to have a beginning.

Also the considerations of this work show how important it is to consider in detail the beginnings of the epistemological process. For it is at the beginning that the problems discussed are most transparent and easy to see. When an observer is far along in the process of acquiring knowledge, it seems "obvious" that the space-time labels of each single measurement in a sequence are irrelevant variables. Also it seems "obvious" that one can know how to construct sequences satisfying the "same" requirements as well as know about basic empirically decidable properties. However, at the beginning of the process where an observer does not have this knowledge, the problems of how to construct and describe acceptable sequences of single measurements become acute¹⁸.

This and other problems discussed here show that it is necessary to discover minimum conditions which a sequence of single measurements must satisfy if it is to yield a limit empirical mean which is suitable as a point of contact between theory and experiment. It has been shown elsewhere⁶ within a probability theory framework, that some minimum conditions of acceptability for a sequence of single measurements are that it be ergodic. That is, the sequence must satisfy an ergodic theorem and be metrically transitive.

However, this immediately brings up the problem of how one is to know that sequences (such as those by which the homogeneity of space-time is tested)

¹⁸ P. A. BENIOFF, Unpublished Work.

are ergodic. In particular, the problem of how one knows they are metrically transitive seems especially vexing. Thus it would appear that the problems arising from how one knows that the "same" requirements are satisfied are transferred to a more basic level and still remain. However, the fundamental ergodic requirements do suggest some new approaches which are currently being investigated.

Another interesting point is that the basic problems of the beginning of the epistemological process suggest that the foundations of this process lie outside of quantum mechanics^{10, 18, 19}. The reason for this is that quantum mechanics makes contact with experiment only through expectation values or limit empirical means. Thus if quantum mechanics is to be used to describe the epistemological process an observer must be able to know expectation values in terms of limit empirical means by any finite time. However, as was seen, quantum mechanics and relativity forbid this knowledge except in the infinite future. Thus only the infinite future, or not at all, could quantum mechanics serve as a basis for the epistemological process.

Another way to see this is that if epistemology were described by quantum mechanics, then any

properties of the single measurements of a sequence which would serve as acceptability conditions, must be given empirically as expectation values. However, this implies other sequences of single measurements and again the infinite regression problems as well as how one is to start the knowledge acquisition process occur. Thus, casting the results of this work into a slightly different form, it appears that use of the quantum mechanics as a foundation for the epistemological process would not allow the process to have a beginning. Since the process does have a beginning its foundation must lie outside of quantum mechanics.

A final point relates to the basic fact of epistemology that an observer can, by any finite time, know only a finite number of empirical results. This fact has usually been considered irrelevant to and outside of physics. For classical mechanics this is indeed the case. However, it is interesting and possibly significant that this fact of epistemology is required by quantum mechanics and relativity. In fact this correspondence is further evidence for the already suggested close relationship²⁰ between quantum mechanics and epistemology.

¹⁹ J. S. BELL and M. NAUENBERG, *The Moral Aspect of Quantum Mechanics in Preludes in Theoretical Physics*, Edited by A. DE SHALIT, H. FESHBACH, and L. VAN HOF, North Holland Publishing Co. Amsterdam 1966, pp. 279—286.

²⁰ E. WIGNER, *Amer. Jour. Phys.* **31**, 6 [1963]. *Rend Scuola Intern. Fis. Enrico Fermi Course 29, Varenna (Italy) 1963*, Academic Press Inc., New York 1964; "Remarks on the Mind Body Question" in *The Scientist Speculates*, Edited by I. GOOD, W. HEINEMANN, London 1962, A. SHIMONY, *Amer. Jour. Phys.* **31**, 755 [1963]; P. A. MOLDAUER, *Ibid.* **32**, 172 [1964]; M. N. HACK, *Ibid.* **32**, 890 [1964].